Chaos, Fractals and Solitons in A Model for Earthquake Ruptures

Elbanna, Ahmed (aettaf@caltech.edu) and Thomas Heaton (heaton@caltech.edu)

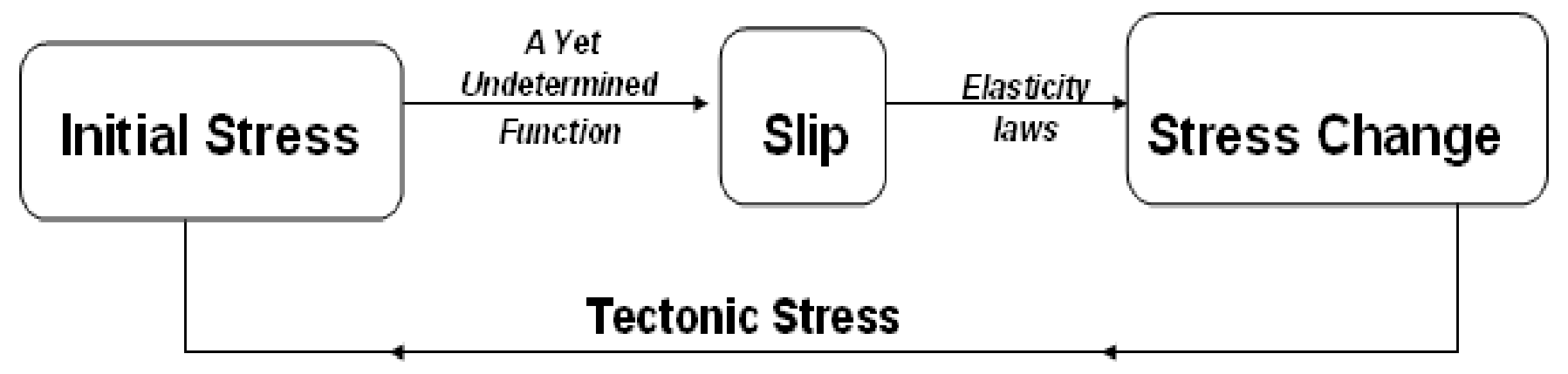
1-Introduction:

One of the most fundamental features of Earthquake ruptures is that they exhibit multi-scale spatial-temporal complexity which suggests chaotic behavior. Understanding chaotic dynamics from spontaneous rupture in a continuum requires: i) Description of the properties of friction in space and time and ii) Description of the pre-stress in space.

Unfortunately, the pre-stress cannot be described in an ad-hoc fashion; the pre-stress must be deduced by allowing the system to evolve to a statistical steady state that is compatible with the friction law. This means that many consecutive multi-scale spontaneous ruptures must be simulated to study the dynamics of a particular friction law.

Simulations of 3D continua with friction laws appropriate for high-speed slip (>1m/s) are numerically intractable with the current computational capabilities. Therefore, it has not been possible to describe the implications of a particular friction law.

In an attempt to solve this problem, we choose to follow a different path. We investigate the possibility of constructing “reduced models” that retain essential physics of the complex rupture process and that facilitate simulation of many cycles of events in a reasonable computational time.



To better understand the spatial-temporal complexity of earthquake ruptures, and to find a clue for the sought “reduced models”, we start by investigating the evolution of dynamics in mechanical models that exhibit analogous features to real ruptures, but which have the advantage that they are numerically tractable over many cycles of events.

In this poster, we show some of the results of our recent work on the study of mechanical models similar to the Burridge-Knopoff spring-block-slider models. We attempt to understand and describe the rich dynamical behavior we found in these systems. We are specially interested in identifying the conditions in the friction law that are required to promote spatial-temporal complexity, as well as in tracking the evolution of stress through multiple cycles of events.

Although there are limitations in the usage of spring-block-sliders to interpret real earthquakes (they do not show the long-range interactions present in continuum models), these models have the advantage that they are computationally efficient. This allows us to explore the nature of complexity that is produced by different types of models of dynamic friction.

2- Chaos in low dimensional models (2-Block Model):

The purpose here is to explore what class of friction laws might lead to chaotic dynamics.

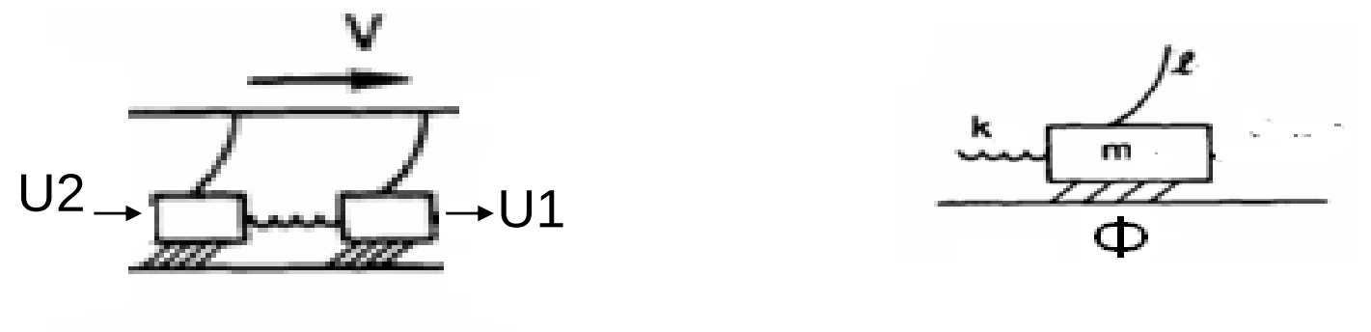


Figure 1. 2-block spring slider. Two rigid half-spaces move steadily at rate V. Point masses slide according to friction law Φ. The blocks are coupled to the loading plate by springs of stiffness (l) and to each other by springs of stiffness (k).

Friction laws considered are functions of either slip or slip rate and they include: i) Hyperbolic laws (no inherent scale), ii) Traditional slip weakening laws (a characteristic weakening distance and slope). These are shown in Figure 2.

Only hyperbolic weakening friction laws led to chaotic behavior for a wide range of weakening rates. *Chaos is manifested i) in the complicated behavior of the blocks' phase space (see Figure 3), and ii) in obtaining events of different magnitudes (sizes) for a sequence of events obtained by running the model for an assumed friction law (see Figure 4). This system does not evolve to chaos through the usual period doubling route.*

Figure 2. Left panel shows hyperbolic friction laws that weaken hyperbolically with slip or slip velocity. These laws have no inherent scale. The right panel shows the traditional slip weakening friction that has an inherent length scale u_c.

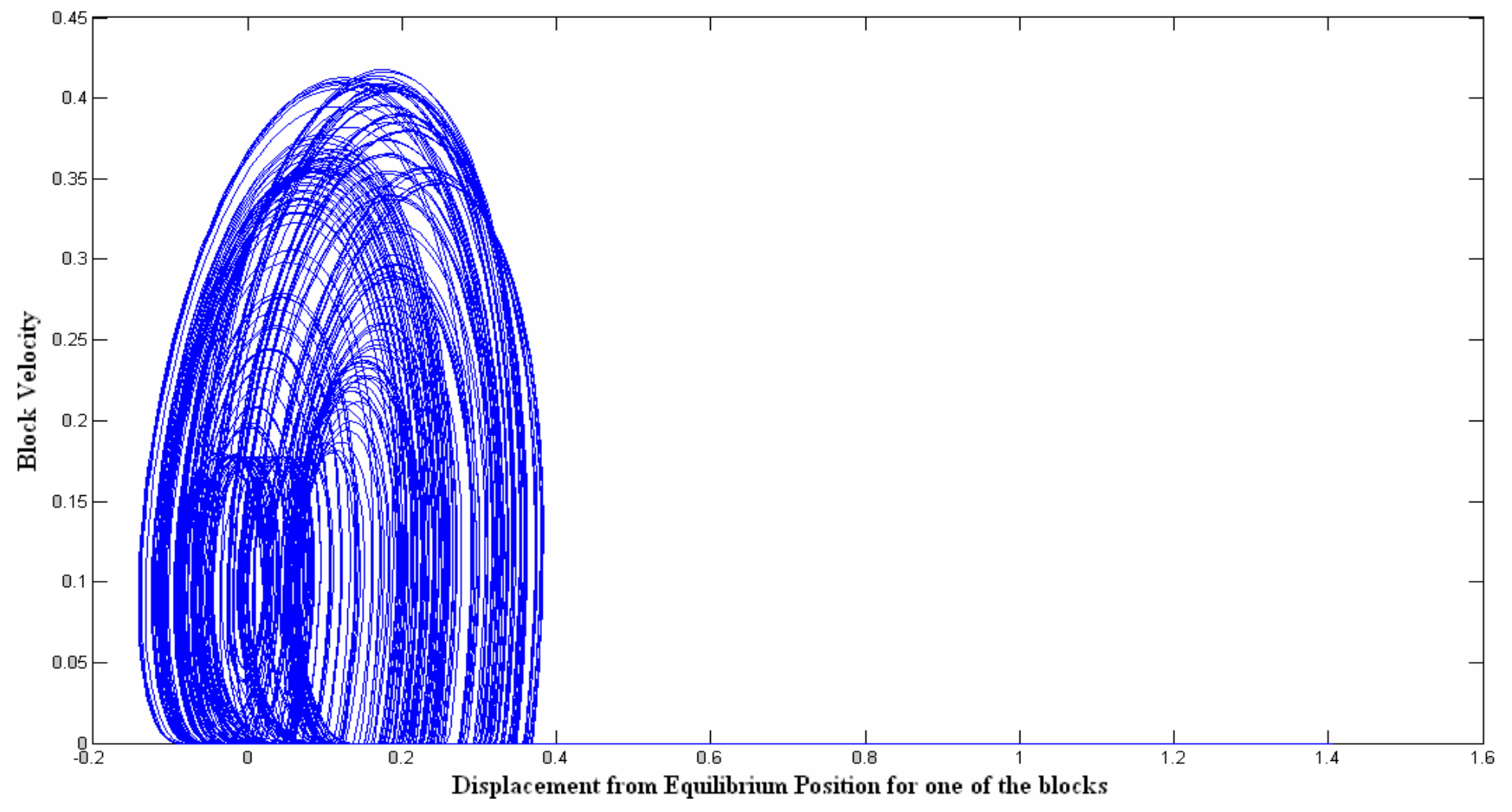


Figure 3. Block velocity vs displacement from equilibrium (phase space plot of one block of the 2-block system for a hyperbolic friction law. Notice that the phase trajectories cover a wide range of the phase space, an indication of chaos.

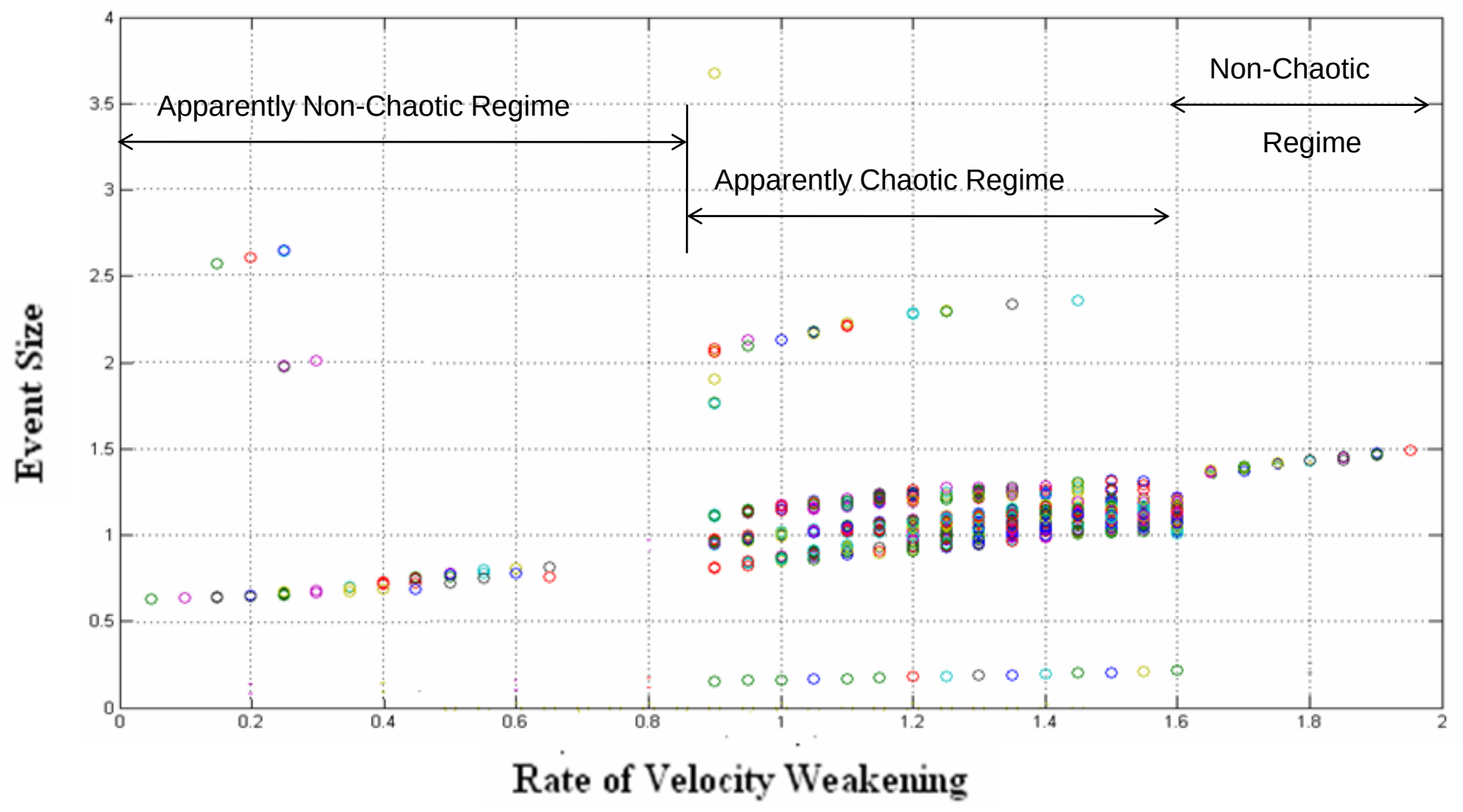


Figure 4. Bifurcation diagram for the size of events versus the rate of weakening in velocity dependent friction laws. Notice that many different sized events occur when the velocity weakening rate is nearly between 0.9 and 1.5, which is a possible indication of chaos.

3- Slip pulses and Solitons in the Multi-Block Model:

We studied the dynamics of a chain of spring-sliders subjected to hyperbolic velocity dependent friction (in the chaotic regime). We obtained events of different sizes (see Figure 5). The contours of particle velocity in one of those events are shown in Figure 6. It is clear from these figures that the prevailing rupture mode is a propagating slip pulse.

Figure 7 shows the propagating pulse in two consecutive times. There is a strong analogy between slip pulses and solitons in the theory of nonlinear PDEs. This suggest that we could use methods from Soliton theory to describe the propagation of slip pulses in our model.

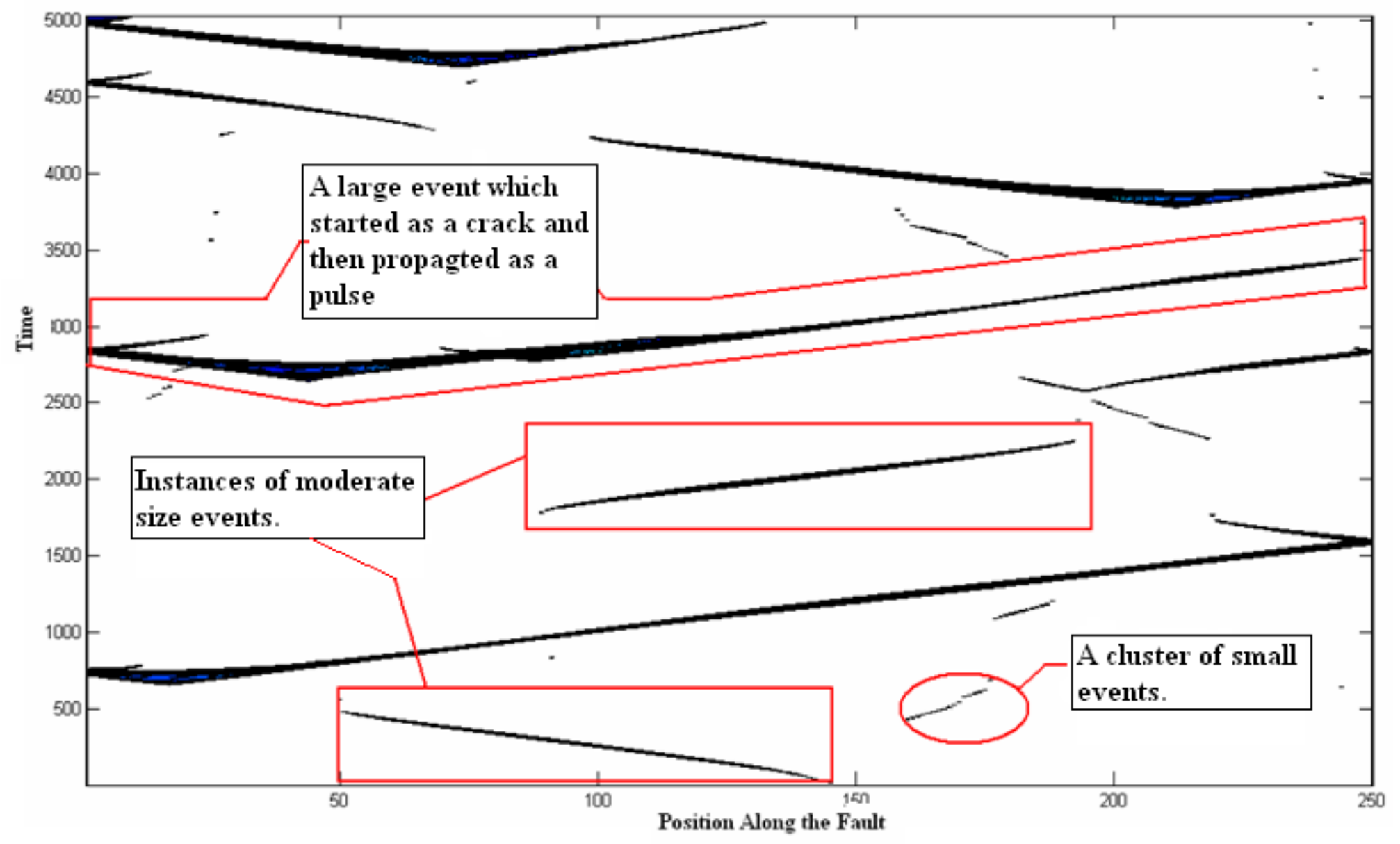


Figure 5. Contours for particle velocity in a number of generated events showing a variety of event sizes as well as existence of slip pulses, since the rupture duration at any of the sliding blocks is small compared to the total rupture time in the corresponding event .

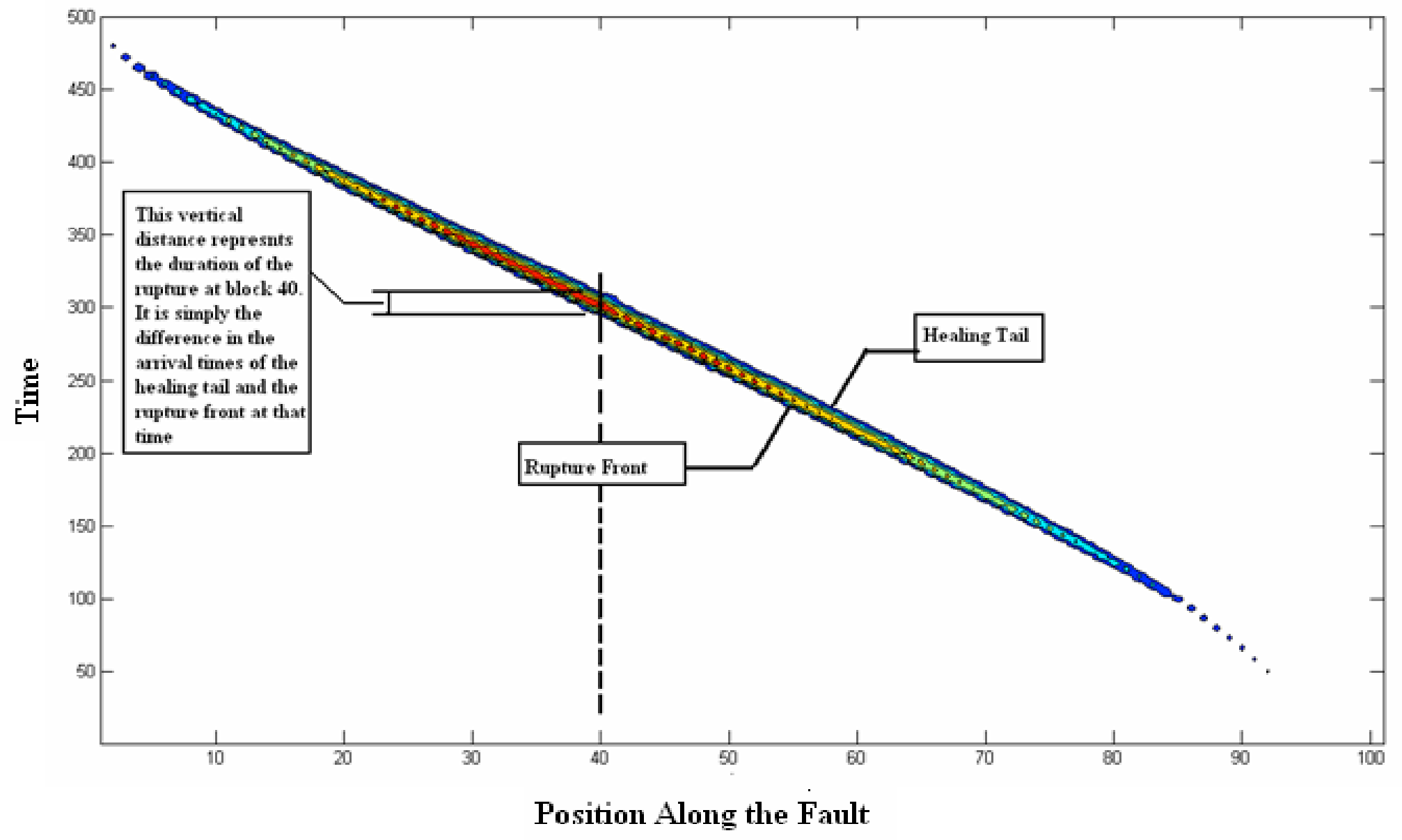


Figure 6. A detailed look at the contours for particle velocity of one of the generated events (the lowermost event in Figure 5). Note that the event consists of a solitary wave of slip (the maximum duration of slip in this event is about 5% only of the total rupture time for the whole event).

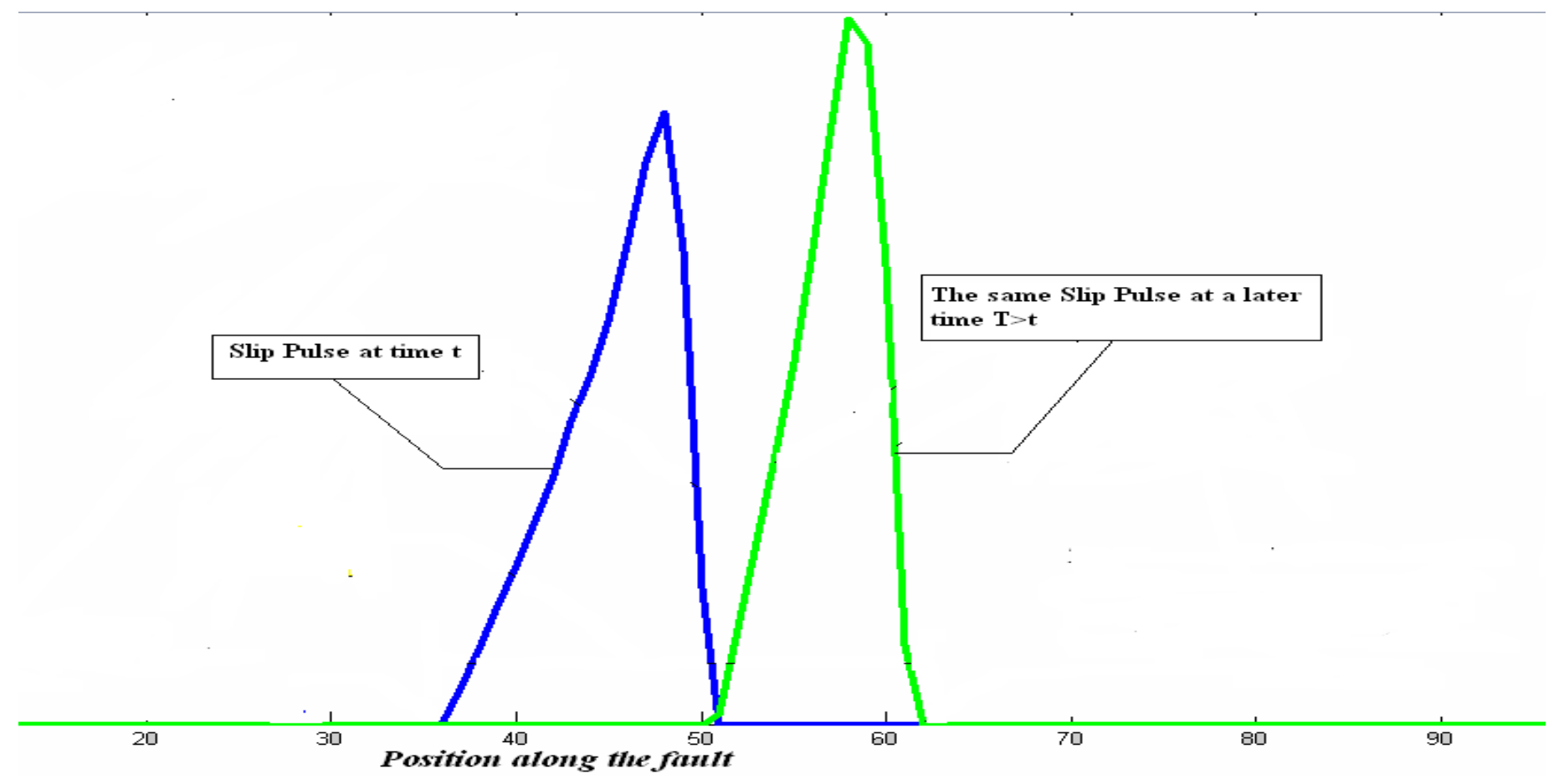


Figure 7. A slip pulse in two consecutive times. They look similar to solitons. Unlike the steady-state solitons known in the mathematical literature, slip pulses change their shape and speed due to interaction with the pre-existing traction on the blocks. Since the theory of solitons is well established, the analogy between slip pulses and solitons suggests that we could use methods for non-steady solitons to formulate models for our propagating pulses.

4- Stress is possibly fractal:

We studied the evolution of stress in the multi-block system when hyperbolic velocity dependent friction is used. Although the initial stress was piecewise constant and the static friction was uniform, this stress evolves into a heterogeneous distribution after few events (see Figure 8).

The wavenumber power spectrum of the evolved stress (Figure 9) shows a power-law dependence, suggesting that the stress might evolve into a fractal distribution.

The variation in the wavenumber power spectrum of the stress from one event to another is shown in Figure 10. We can infer from this figure that, as the number of blocks increase, this variation decreases and possesses longer wavelengths. We speculate that, in the continuum limit and in the limit of long time, we would reach a statistical steady state.

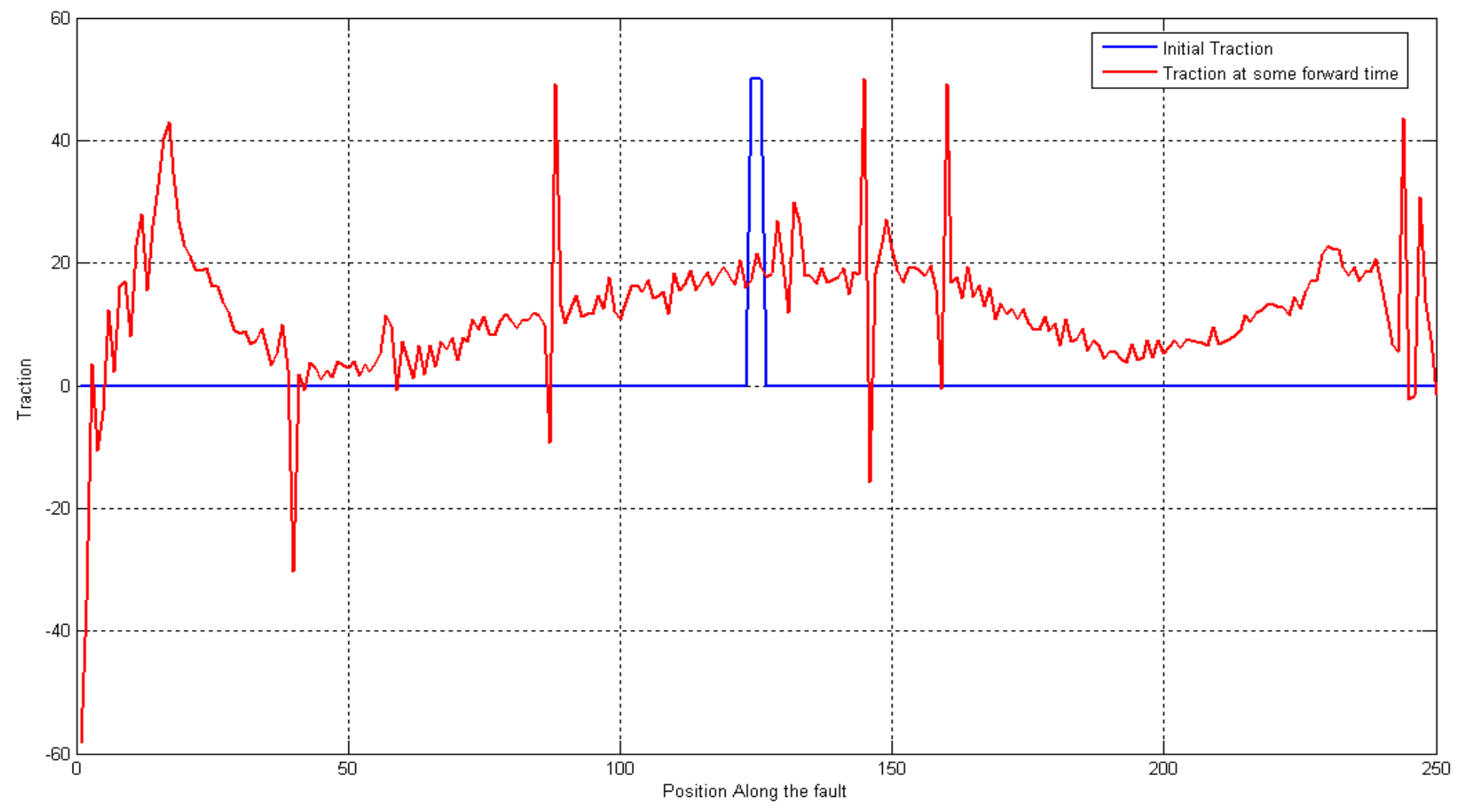


Figure 8: Evolution of stress in the multi-block model. When hyperbolic velocity weakening friction is used, the initial traction (shown in blue) evolves into the heterogeneous distribution shown in red after 120 events, even though the static friction was assumed to be spatially uniform.

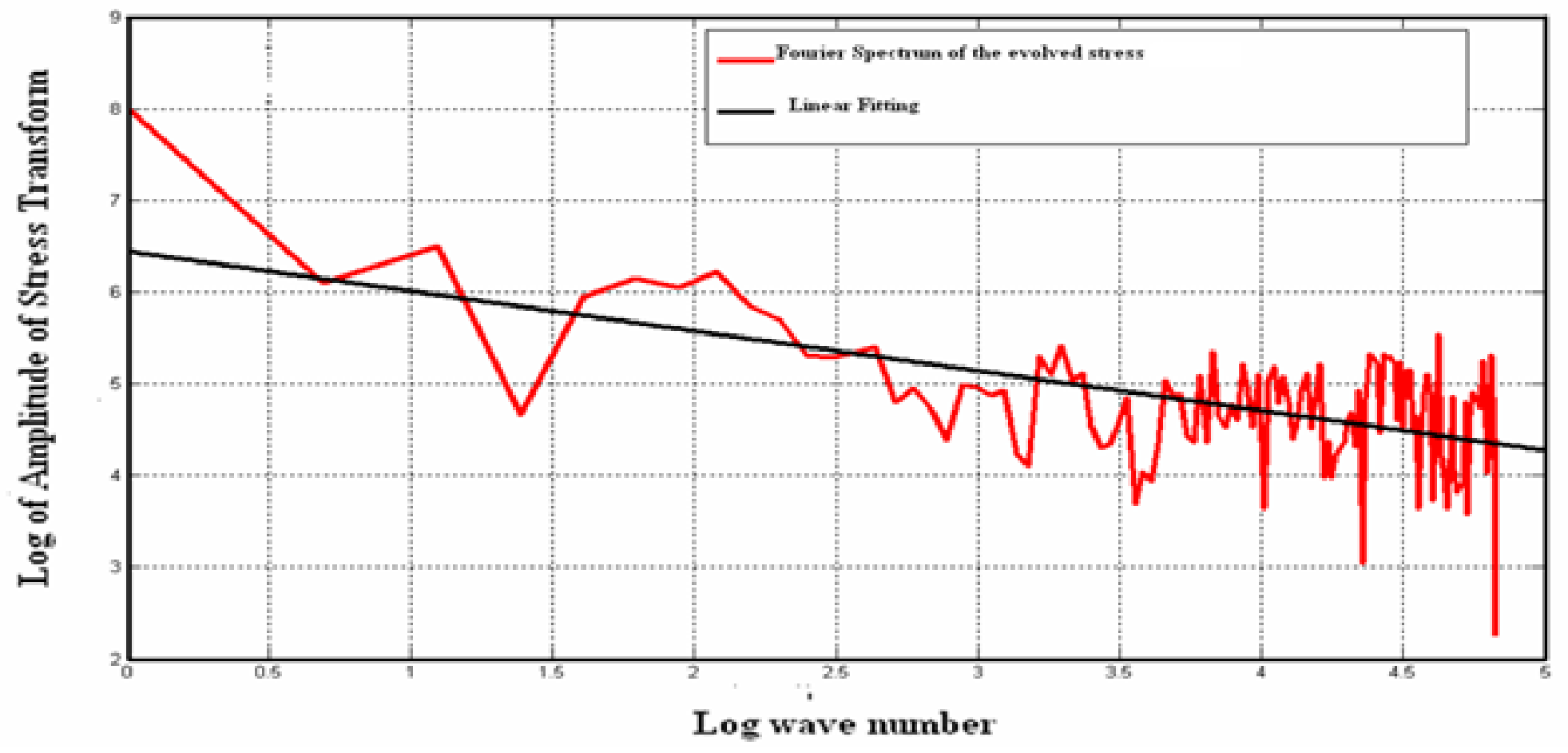


Figure 9. The wavenumber power spectrum of the evolved stress shown on a log-log scale. The spectrum could be fit to a straight line showing that it has a power-law dependence. This is suggestive that the stress might evolve into a fractal distribution if scaleless friction laws are used (e.g. hyperbolic velocity dependent friction)

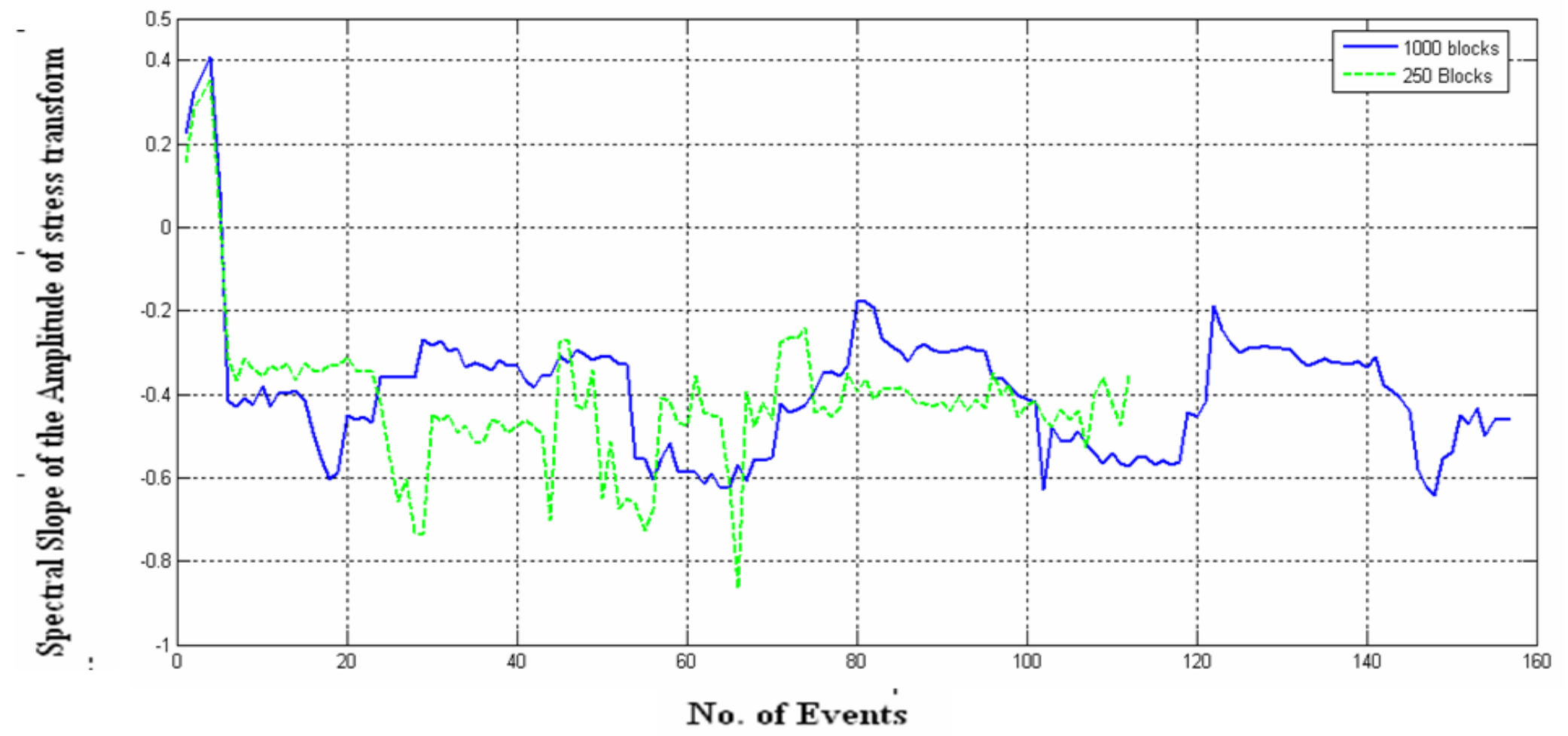


Figure 10. Variation in the power law coefficient used to describe the wavenumber spectrum of the evolved stress as the number of events increase. Green and purple are for 250-block and 1,000-block systems, respectively. Increasing the number of blocks better resolves the stress heterogeneity which in turn makes the variation in the power law coefficient decrease and exhibit longer periods.

5- Conclusions:

1.Friction laws that have no characteristic scale, such as hyperbolic slip- or slip-velocity dependent friction laws, promote chaos even in very low dimensional systems like the 2-block systems.
2.Traditional slip weakening laws cannot lead to complexity; they result in repeating characteristic events.
3.The hyperbolic friction laws generate slip pulses and lead to stress heterogeneity, even if the static friction and the initial stress are uniform.
4.The wavenumber spectrum of the stress after a few number of events exhibits a power-law dependence. This suggests that the stress might evolve into a fractal distribution if the model is run for a long enough time.
5.As the number of blocks in our model increases, the variation in the power law coefficient of the stress wavenumber spectrum decreases and exhibits longer wavelengths. We speculate that, in the continuum limit and after a long enough time, the system should settle into a stable statistical state.