

A Time-Reversed Reciprocal Method for Detecting High-Frequency Events in Civil Structures with Accelerometer Arrays

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ABSTRACT

A high-frequency experimental method of detecting a failure event in engineered structures is presented that uses the property of wave propagation reciprocity and time-reversed reciprocal Green's functions. The premise is that if a numerical database of pre-event, source-receiver Green's functions can be compiled for multiple locations of potential damage in a structure, that database can subsequently be used to identify the location and time of occurrence of a real failure event in the structure. Once a fracture source emits a wavefield that is recorded on a distributed set of accelerometers in the structure, time-reversed waves can be obtained by convolving the displacements with the database of time-reversed Green's functions and stacking the results. The correct location and time of the fracture source can be inferred from the subset of Green's functions that exhibits the best focus in the form of a delta function. The 17-story, steel moment-frame UCLA Factor building contains a cutting-edge, continuously recording, 72-channel, seismic array. The accelerometers' 500 sample-per-second recordings have been used to verify the ability to observe impulse-like sources in a full-scale structure. Application of an impulse-like source on the 3rd and 15th floors of the Factor building shows that the associated displacements serve as useful approximations to the building's Green's functions in the far field, and can be used in investigations of scenario fracture location and timing.

INTRODUCTION

Understanding the data signature of building damage due to earthquake shaking is fundamental to designing structural health monitoring and damage detection tools. The 1994 Northridge, California earthquake highlighted a common type of structural failure, fractured welds in beam-column connec-

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tions, that is difficult to identify either visually or through localized ultrasonic testing (Roeder, 2000). Weld fracture significantly decreases the ductility of tall steel buildings (Hall et al., 1995). The prevalence of fractured welds in mid-rise and high-rise structures shows how new computational tools are needed to immediately identify their occurrence after a large earthquake for the health of the structure and the safety of the occupants.

Dense structural networks are producing orders of magnitude more data than before; thus it is becoming more challenging to wade through the volumes of data to determine whether something unique has happened and where in the structure it has happened. There is a need for new computational techniques that will enable end-users to identify and locate (spatially and temporally) significant damage events. Since uncertainty is intrinsic in any analysis of this type, it will also be necessary to develop robust methodologies for error estimation. Predictive numerical simulations will help to determine just how well structural array data can resolve the location of damage. Advances in SMART structure embedded sensor networks and sophisticated in-network processing are making damage assessment techniques, such as that introduced here, application-realistic.

We present a new, high-frequency numerical method of detecting a failure event in engineered structures that uses the property of wave propagation reciprocity and time-reversed reciprocal Green's functions. The focus here is on a specific type of structural failure event: brittle-fractured welds of beam-column connections. In this study, we postulate that if we can compile a numerical database of pre-event, source-receiver Green's functions for multiple locations of potential damage in a structure, we should be able to use that database to identify the location and time of occurrence of a subsequent, real failure event in the structure. Once a fracture source emits a wavefield that is recorded at a distributed set of accelerometers in the structure, time-reversed waves can be obtained by convolving the recorded displacements with the database of time-reversed impulse-source Green's functions. The pre-event impulse sources can be, for example, hammer blows applied adjacent to the welded connections, ideally during construction when they are still accessible. The correct location and time of each subsequent fracture source can be inferred by identifying which Green's function most nearly produces a delta function when it is time-reversed and convolved with a record from an individual station. Time-reversed convolution of two time series is, alternatively, cross correlation. If an actual weld fracture is seen in several stations, it should produce an impulse at the same time for the cross correlation at each station. By summing (stacking) the cross correlations from several stations, it should be possible to significantly improve the resolution of the technique.

This approach assumes that we have a general idea of where the failure event (i.e., the fractured weld) is likely to occur based on the known weld locations, and that a multi-channel seismic network will record the event. The number of structures with dense seismic networks embedded in them is increasing; thus there is an opportunity for new approaches to identifying damage that take advantage of high-density processing techniques such as stacking and beamforming for significantly increased signal-to-noise ratios. The expense and permitting difficulties associated with installing permanent seismic arrays are rapidly becoming surmountable with the growing popularity of deploying cheap, easy-to-install, MEMS-based USB sensors at desktop computer sites.

Whereas traditional modal analysis typically used in system identification techniques theoretically does not need input data from more than one sensor to detect perturbations in modal frequencies, characterization of the complete wavefield from an internal (e.g., weld fracture) or external (earthquake) source of vibration requires spatial sampling on the scale of twice the smallest wavelength desired. The development of robust designs in seismometer hardware and software is making it more feasible to densely instrument civil structures on a permanent basis in order to study their states of health.

The 17-story UCLA Factor building (Fig. 1) contains one of those cutting-edge structural arrays, recording building vibrations at high sample rates. The array consists of 72 single-channel accelerometers recording continuously on 24-bit digitizers. It is one of only a handful of buildings in the U.S. permanently instrumented on every floor, providing information about how a common class of urban structures, mid-rise moment-frame steel buildings, will respond to strong ground shaking (Kohler *et al.*, 2007). Structural stiffness undoubtedly decreased when welded, beam-column connections fractured

extensively in numerous moment-frame steel buildings during the 1994 Northridge earthquake. Unfortunately, there are very few seismic records from buildings with this type of damage. Ample experimental evidence motivates an in-depth study on how changes may be observable through analysis of vibration data for an instrumented building.

THEORY

The purpose of this paper is to develop a method which illustrates how wave propagation properties could be used in a new approach to identify structural damage events in a structure. We are motivated by experimental evidence that such waves will be readily observable on strong-motion networks by understanding what features to look for in high-sample-rate data. Our strategy is to detect characteristic high-frequency waveforms produced at each station by fracture of any individual weld. Since we cannot actually fracture all the welds in a building to produce these waveforms, we explore the possibility of using data from hammer blows adjacent to welds as proxy sources.

We first discuss the similarities and differences of high-frequency waves expected from two types of sources: 1) an impulsive point force due to a hammer blow, and 2) a near-instantaneous tensile failure of a welded connection. In particular, we are interested in the differences in the waveforms that will be produced by these two sources. By definition, the displacement from the hammer blow, u^H , is described by the Green's function for the building, or

$$u_i^H(\mathbf{x}, t) = F_j(\xi, \tau) * G_{ij}(\mathbf{x}, t - \tau; \xi, 0) \quad (1)$$

where the observed i^{th} component of displacement occurs at receiver location \mathbf{x} , and the source occurs at location ξ at time τ (e.g., Aki and Richards, 1980). G_{ij} is the Green's function for the i^{th} component of displacement due to an impulse source point force F in the j^{th} direction for a specific source-receiver path.

The weld fracture problem is more complex. In principle it could be simulated using an elastic model with a tensile crack that experiences a step change in normal traction on the crack surface. However, this is a difficult problem to solve analytically and we can gain important insight by approximating the weld fracture as a localized region that experiences very large elastic tensile strains, thereby causing finite opening across the strained region. This is a body-force equivalent source that is often used in seismology. The source is characterized by the seismic moment tensor which is essentially a point stress multiplied by the product of the crack area times the crack opening (Aki and Richards, 1980). Unlike a point force, the moment tensor consists of combinations of linear force couples (the diagonal components) and shear force couples (the off-diagonal components). The response of the medium to these force couples is just the spatial derivative of the point force Green's functions, or

$$u_i(\mathbf{x}, t) = M_{jk}(\xi, \tau) * \frac{\partial}{\partial \xi_k} G_{ij}(\mathbf{x}, t - \tau; \xi, 0). \quad (2)$$

The body force equivalent source for a point tensile crack is given by Burridge and Knopoff (1964) as a point moment tensor whose amplitude is

$$\mathbf{M}_0(t) = SD \begin{bmatrix} \lambda + 2\mu & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \quad (3)$$

where S is the area of the tensile crack, D is the average opening, λ is the first Lamé constant and μ is the rigidity. Since we are describing the size of a singular strain that results from a singular point stress, the solution only depends on the elastic constants at the point of application of the stress. We have assumed that the crack opens instantaneously and hence the time history of crack opening is assumed to be a Heaviside step function. Since everything in the problem is linear, any opening time history can be achieved by convolution with the appropriate time function.

We will show, however, that for a broad, applicable set of fracture recording conditions, the impulsive force Green's functions serve as useful approximations to their spatial derivatives, but with a step function time history,

$$u_i(\mathbf{x}, t) = M_{jk}^0 H(\tau) * \frac{\partial}{\partial \xi_k} G_{ij} \quad (4)$$

$$= M_{jk}^0 \int_{-\infty}^{\infty} \frac{\partial}{\partial \xi_k} G_{ij}(\mathbf{x}, t - \tau; \xi, 0) d\tau. \quad (5)$$

The point source is treated here as a general source. For example, a weld fracture at or near the steel beam-column connection is likely to be a combination explosive plus single force couple mechanism. Bolt slippages would be analogous to strike-slip double couple sources.

For any point source in an elastic medium, the response of the medium can be decomposed into near-field and far-field terms. We will use this separation to show that for recording conditions in a building, we can use G as an approximation for the time integral of its spatial derivative. The near-field terms generally comprise the particular solution to the equation of motion. Near-field terms are necessary to describe any static displacements that result from the source (i.e., a steady-state solution). Although their time history can be complex, their time dependence is generally given by that of the point source, or definite time integrals of the time history. The distance decay of near-field terms is typically the same as, or faster than, the decay of the static solution.

The rest of the solution is described as far-field terms. These far-field terms are solutions to the homogeneous equation of motion and they can be viewed as the free vibrations of the system. The far-field terms generally have a time history that is a time derivative of the time history of the near-field terms. In the absence of significant damping, they typically persist indefinitely with equipartition of potential and kinetic energy. The far-field terms can always be decomposed as either a sum of normal modes, or as a sum of traveling waves. If we consider the Green's function to be a sum of rays, each traveling at some velocity c_ℓ , then we can approximate the far-field portion of the Green's function (the large distance, high-frequency part of the solution) as

$$G_{ij}\left(\mathbf{x}, t - \tau; \xi, \tau > \frac{r}{c}\right) \approx \sum_{\ell=1}^{\infty} A_{ij\ell} \delta\left(\tau - \frac{r_\ell}{c_\ell}\right) \quad (6)$$

where A are the amplitude coefficients corresponding to each ray ℓ (e.g., P- and S-waves traveling different paths through the frame) and r_ℓ signifies the path length that the ray takes to get from the source to the receiver. This approximation is an oversimplification of the dynamics of a real building where various wave types may be dispersive (e.g., bending waves in a floor slab). Furthermore, there may be parts of the solution that are difficult to represent as rays. Nevertheless, this analysis shows why weld fractures and hammer blows have far-field wavefields that are similar in nature. Considering only the radial portion of the gradient operator, Eqs. (5) and (6) can be rewritten as

$$u_i(\mathbf{x}, t) \approx M_{jk}^0 \int_{-\infty}^{\infty} \frac{\partial}{\partial \xi_k} \left[\sum_{\ell=1}^{\infty} A_{ij\ell} \delta\left(\tau - \frac{r_\ell}{c_\ell}\right) \right] d\tau \quad (7)$$

$$= M_{jk}^0 \int_{-\infty}^{\infty} \left[\sum_{\ell=1}^{\infty} A_{ij\ell} \delta'\left(\tau - \frac{r_\ell}{c_\ell}\right) \left(-\frac{1}{c_\ell} \frac{\partial r_\ell}{\partial \xi_k}\right) \right] d\tau \quad (8)$$

$$= M_{jk}^0 \left[\sum_{\ell=1}^{\infty} B_{ij\ell} \int_{-\infty}^{\infty} \delta'\left(\tau - \frac{r_\ell}{c_\ell}\right) \right] d\tau \quad (9)$$

$$= M_{jk}^0 \sum_{\ell=1}^{\infty} B_{ij\ell} \delta\left(\tau - \frac{r_\ell}{c_\ell}\right). \quad (10)$$

We see that we expect similarities between the seismograms produced by a hammer blow point source (Eq. 6) and the seismograms produced by a weld fracture (Eq. 10) since they are only different by the ratio of the constants A and B . Ideally we would need to compute $\partial G_{ij}/\partial \xi_k$ to obtain the appro-

appropriate displacement response for a fracture source. However, Eq. (10) shows that the impulse source G_{ij} serves as a useful approximation since it is different from $\partial G_{ij}/\partial \xi_k$ only by a constant of proportionality in this approximation. Of course, the problem is more complex in detail and a careful comparison requires numerical simulation of hypothetical buildings experiencing impulse force and opening crack source mechanisms.

OBSERVATIONS OF POINT FORCES AND FRACTURES

If the excitation is an impulse that can be described by a point force in the j^{th} direction $F_j(\xi, \tau) = F_j^0 \delta(\xi) \delta(\tau)$, what might the Green's functions look like recorded on an actual building network? In April, 2007, we conducted an experiment to record responses to an impulse-like source on the 3rd and 15th floors of the UCLA Factor building. The goal was to test whether a single, high-frequency impulse source would be observed throughout the building; the data illustrate that, indeed, it is. We applied a hammer blow to a steel plate on the concrete floor of a corner of the 15th floor and in a stairwell in the SW corner of the 3rd floor. The data recorded from the 3rd floor hammer blow is shown in Fig. 1. The 15th floor hammer blow waveforms are comparable, with the top floors exhibiting the earliest arrivals and highest signal-to-noise ratios. The blow was nearest the accelerometers on the south and west walls of the 2nd and 3rd floors. These waveforms are the responses recorded on the 500 sps streams of the Factor array; however they have been filtered for frequencies between 10 and 95 Hz in order to compare with 200 sps fracture data presented in the next section. Maximum acceleration was 0.006 g. Fig. 1 shows the resulting traveling wave throughout much of the building. Top floors typically have higher background noise, eventually obscuring the signal. The travel time from the 3rd floor to the top of the building for this frequency range is ~ 0.05 s, corresponding to an average wave speed of 1200 m/s. Opposite floor arrivals are delayed due to horizontal wave propagation through slower concrete floor slabs.

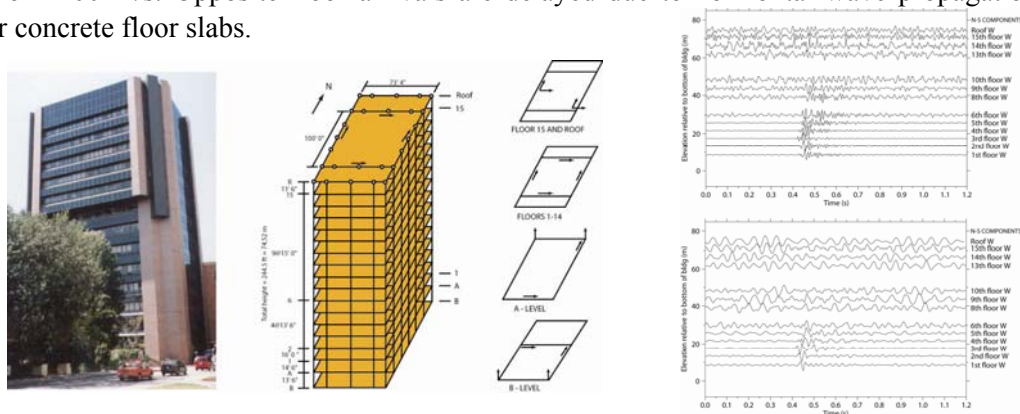


Fig. 1. **Left:** The UCLA Factor building and its seismic array. Arrows show locations and polarities of sensors on each floor. **Right:** Accelerations (top) and displacements (bottom) recorded from force hammer impulse source applied near 3rd floor SW corner in the Factor building.

To demonstrate what an actual fracture would look like on typical strong-motion instrumentation, we recorded data from an induced fracture that occurred during a beam-column steel connection test for a commercial steel manufacturer. The test was conducted at UC San Diego's Department of Civil Engineering. Cyclical loading was applied to a full-scale, steel beam-column to test its brittle-plastic response. Prior to the test we installed a $\pm 2g$ Episensor at one end of the beam and connected it to a 24-bit digitizer. The initial sensor orientations were horizontal along the length of the beam (X direction), horizontal and perpendicular to the beam (Y direction), and vertical (Z direction). We recorded waveform data at 200 sps throughout the test. Cyclical loading corresponding to story drifts between 1% and 7% was applied at one end of the beam by a hydraulic piston. The test ended at 7% story drift when a crack occurred at the bolts near the welded beam-column connection. Interestingly, the crack did not actually occur in the weld but at the bolts which experienced large plastic strains and eventual embrittlement due to strain hardening.

The accelerometer recorded both bolt stick-slip and actual fracture events on scale (Fig. 2), driving home the fact that such failures in real structures will be recorded on-scale on modern equipment, and are observable because their amplitudes are much larger than other sources of vibration. Maximum acceleration from the fracture was 1.5 g. To predict what wave propagation might look like in the Factor building from a scenario fracture source suggested by the data, we convolved the hammer source displacement pulses with the X-direction fracture displacement. The predicted response exhibits the dramatically increased signal-to-noise ratios observed in the beam fracture data (Fig. 2).

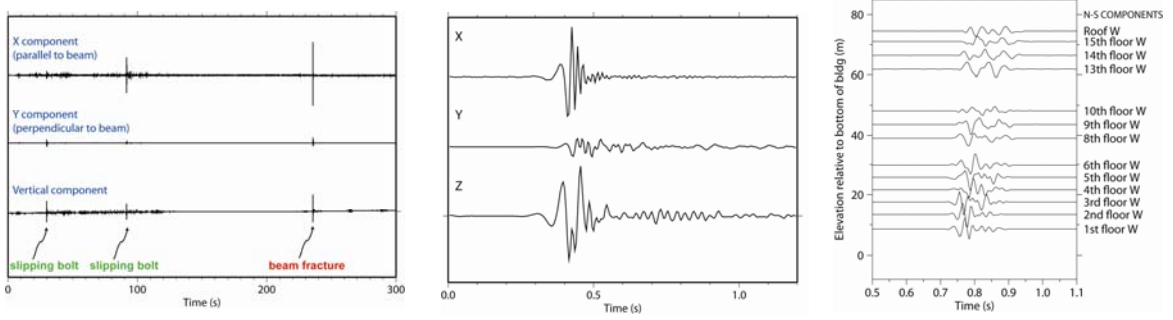


Fig. 2. **Left:** Accelerations recorded during moment-frame connection test. **Middle:** Blow-up of displacements at time of fracture. **Right:** Predicted response of Factor building to scenario fracture source.

RECIPROCITY AND SOURCE IMAGING

In order to better understand the specific scheme that we are proposing to detect and locate weld fractures, it is helpful to first review the concept of source-receiver reciprocity. Because the system is linear, we can always exchange source and receiver locations (i.e., put our impulsive force at an accelerometer location and vice versa) and obtain the same Green’s function. Space-time reciprocity dictates that

$$G_{ij}(\mathbf{x}, t - \tau; \boldsymbol{\xi}, 0) = G_{ji}(\boldsymbol{\xi}, t - \tau; \mathbf{x}, 0) = \overline{G}_{ji}(\boldsymbol{\xi}, -\tau; \mathbf{x}, -t) \quad (11)$$

where \overline{G} is the time-reversed Green’s function for the source-receiver path (e.g., Aki and Richards, 1980). This also means that if the source and receiver positions are interchanged, identical seismograms u and v would be recorded at the two sites, except that waves would be traveling either forward from the original source or backwards from the new “virtual” source (old receiver) (e.g., Derode et al., 1995). We conclude that

$$v_j(\boldsymbol{\xi}, \tau) = \overline{G}_{ji}(\boldsymbol{\xi}, -\tau; \mathbf{x}, -t) * F_i(\mathbf{x}, t) \quad (12)$$

where v is the displacement that corresponds to the new receiver (original source) location and time.

In an ideal experimental set-up where a well-distributed structural network has recorded one or more weld fractures and their locations are obtained through waveform or travel-time inversion, we could invert for M_{jk} given numerically or empirically determined G_{ij} . This type of waveform inversion is commonly done for 3D source characterization of cracks in elastic media to identify the kinematic source mechanisms. However, it does not allow for the identification of the source location if the structure’s distribution of physical properties is not known, for example through a finite-element stiffness/mass model. For this we turn to the time-reversed reciprocal Green’s method and stacking techniques to determine the source location. We take advantage of the fact that we can use the impulse source G_{ij} instead of $\partial G_{ij} / \partial \xi_k$ in the far-field approximation (Eqs. 6 and 10).

When a fracture emits a wavefield that is recorded at a distributed set of sensors in the building, those displacements can be treated as secondary sources of energy that are retransmitted back to the original true fracture source. If we know all the pre-damage-event G_{ij} corresponding to a complete set of potential damage locations in a linear structure, we can use forward modeling to compute a range of v to identify the correct $\boldsymbol{\xi}$ and τ , from the suite of u that recorded damage events. Each recorded displacement acts as a secondary point source of waves that, summed, will collapse back in time and space to the correct original source if the correct set of time-reversed Green’s functions is used. The

recorded displacements, acting as the distributed source, are summed to define the excitation force

$$F_i(\mathbf{x}, t) = \sum_{w=1}^W C_w u_i(\mathbf{x}_w, t) \quad (13)$$

(Fink, 1997; Larmat *et al.* 2006). W is the total number of receivers that recorded the fracture event, \mathbf{x}_w are the receiver positions, and C_w is an amplitude coefficient that allows for receiver weighting.

Upon exchanging source and receiver locations, the time-reversed reciprocal expression for displacement in the far-field approximation is then

$$v_j(\xi, \tau) = \int_{\tau}^T dt \iiint_V F(\mathbf{x}, t) \bar{G}_{ji}(\xi, -\tau; \mathbf{x}, -t) dV \quad (14)$$

$$= \sum_{w=1}^W [C_w u_i(\mathbf{x}_w, t) * \bar{G}_{ji}] \quad (15)$$

where v_j is the j^{th} component of displacement at ξ that corresponds to the new receiver (original source) location and time. The waveforms are windowed and filtered for the finite time interval τ to T . Ideally, a large number of azimuthally distributed receivers would produce the best time-reversed wavefield. Earthquake source imaging studies have shown that even a limited number of receivers with associated receiver weights that account for uneven distribution can produce meaningful results (Larmat *et al.*, 2006).

When the displacements containing the fracture signal (Fig. 3b) are individually convolved with the time-reversed impulse source Green's functions (Fig. 3a) and then stacked, the result is the time-reversed wavefield. We infer the source location and time from those values corresponding to the stack that exhibits the maximum amplitude or "best" focus. Stacking the v using the correct set of \bar{G} from a specific fracture location will result in an approximate delta function at $t=\tau$, corresponding to maximum constructive interference (Fig. 3c). The stack adds most coherently for the Green's functions that correspond to the closest fracture location because they will be approximately in phase. The stack should grow increasingly less focused moving away from the location of radiation because the Green's functions are becoming less and less coherent (Fig. 3d).

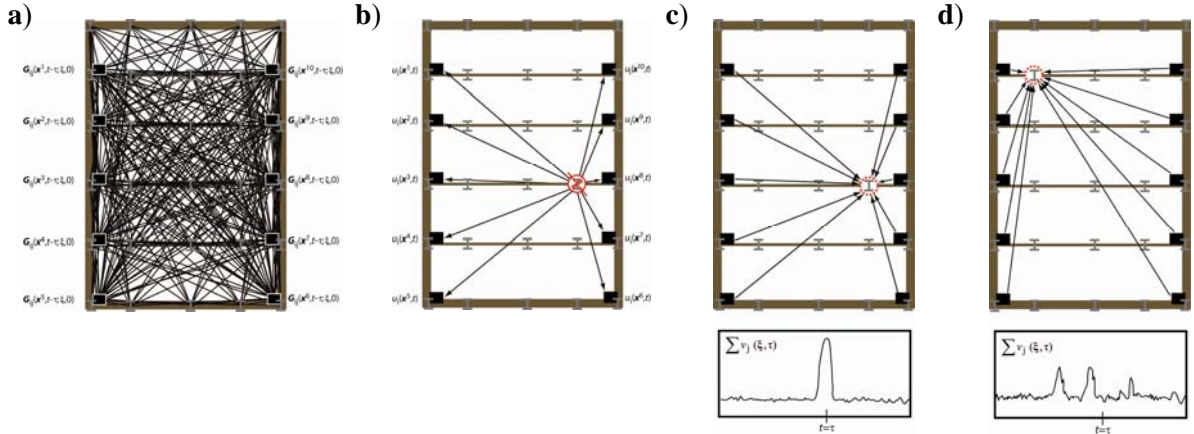


Fig. 3. Schematic diagrams showing: **(a)** Source-receiver paths for impulse source produced at numerous possible weld fracture locations for Green's functions. **(b)** Fracture subsequently occurs at a connection, recorded by all receivers. **(c)** Displacement at *correct* fracture location produced by back-projecting Green's functions for correct subset of source-receiver paths. **(d)** Displacement at *incorrect* fracture location by back-projecting Green's functions for another subset of source-receiver paths.

This is more conveniently approached as a cross-correlation problem in which maximum values are obtained by cross correlating discrete u with the correct set of discrete G_{ij} . Whereas convolution of discrete time series involves the cumulative product of the time-shifted \bar{G}_{ji} and each u , cross correlation can be found as the cumulative product of G_{ij} and u . Maximum correlation occurs when G_{ij} and u are least time-shifted relative to each other. The algorithm would involve systematically running

through the catalogued impulse source Green's functions, each with a known source location, to determine if the cross correlation stack results in a well-resolved delta function around an initially unknown source time $t=\tau$. Ideally, the stations should be evenly distributed and dense enough to fully record the original wavefield. This will never happen in reality, and the station amplitude factor can be modified to weight stations differently depending on their configuration, and to account for wave polarity changes.

Calculating the time-reversed Green's functions and convolving with a set of displacements is straightforward but in the cross correlation computations, time reversing is not necessary. In practical application to structures, it remains to be shown in a numerical study if such a tool could be implemented on structural networks for damage detection. Showing this problem numerically requires modeling frequencies up to at least 200 Hz. Showing how the time-reversed reciprocal Green's functions method works in a 3D finite-element model of a building is the subject of a future study.

CONCLUSIONS

Wavefield properties of linear elastic media provide the basis for a method to determine the location and time of the occurrence of a high-frequency fracture event in a civil structure. The method makes use of long-established principles of wave propagation reciprocity and the properties of time-reversed reciprocal Green's functions. The development presented here has focused on both point force sources and more generalized source mechanisms. We show that for our application, the general Green's functions can be used as an approximation to their spatial derivatives for receivers in the far field. The viability of the technique will be determined through tests using both numerical simulation and experiments on a reduced-scale laboratory structure.

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