Characterizing average properties of Southern California ground motion envelopes

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Abstract

We examined ground motion envelopes of horizontal and vertical acceleration, velocity, and filtered displacement recorded within 200 km from Southern California earthquakes in the magnitude range $2 < M \leq 7.3$. We introduce a parameterization that decomposes the observed ground motion envelope into P-wave, S-wave, and ambient noise envelopes. The body wave envelopes are further parameterized by a rise time, a duration, a constant amplitude, and 2 coda decay parameters. Each observed ground motion envelope can thus be described by 11 envelope parameters. We fit this parameterization to 30,000 observed ground motion time histories, and develop attenuation relationships describing the magnitude, distance, frequency band, and site dependence of these 11 envelope parameters. We use these attenuation relationships to study 1) magnitude-dependent saturation of peak amplitudes on rock and soil sites in various frequency bands, 2) magnitude and distance scaling of P- and S-waves, and 3) the reduction of uncertainty in predicted ground motions due to the application of site-specific station corrections. We compare our envelope amplitude attenuation relationships with amplitude levels predicted by strong motion attenuation relationships in the literature.
Introduction

The widespread deployment of seismic stations in Southern California under the TriNet project has resulted in an unprecedented dataset of recorded ground motions. We analyzed a large portion of this dataset (consisting of horizontal and vertical acceleration, velocity, and displacement ground motion envelopes recorded at Southern California Seismic Network (SCSN) stations located within 200 km of 70 local events in the magnitude range M2.0 through M7.3) as part of a study on seismic early warning (Cua, 2005). We studied envelopes of ground motion, as opposed to the fully sampled ground motion time histories, due to our interests in seismic early warning; ground motion envelope amplitudes, which by our definition are the peak ground motion values over one-second windows, are the data streams that arrive at the central processing facility of the Southern California Seismic Network located at Caltech in closest to real-time. We developed a parameterization that decomposed the observed ground motion envelope time history into P-wave, S-wave, and ambient noise envelopes. Each of the body wave envelopes is described by a rise time, a peak amplitude, a duration, and two decay parameters. With this parameterization, the evolution of ground motion amplitude with time is described by 11 envelope parameters (5 P-wave parameters, 5 S-wave parameters, and 1 constant to describe ambient noise levels). The neighborhood algorithm is used to find the set of 11 maximum likelihood envelope parameters for each envelope history in the database. We then developed attenuation relationships describing each of these 11 envelope parameters as a function of magnitude, distance, site condition, and frequency band. In this paper, we focus the discussion on the attenuation relationships for peak P- and S-wave amplitudes of horizontal and vertical ground motion acceleration, velocity, and filtered displacement on rock and soil sites. We use these attenuation relationships to study 1) magnitude-dependent saturation of peak amplitudes on rock and soil sites in various frequency bands, 2) magnitude and distance scaling of P- and S-waves, and 3) the reduction of uncertainty in predicted ground motions due to the application of site-specific station corrections. We found evidence of saturation at close distances to large events of peak ground motions. This saturation appears to be a source effect at the mid-to low-frequency range. At higher frequencies, saturation of ground motion amplitudes may be indicative of nonlinear site response. We also found that P-wave amplitudes appear to exhibit stronger saturation characteristics than S-wave amplitudes, particularly in the horizontal direction. In general, our attenuation relationships give comparable ground motion levels as those from other strong motion relationships in the literature.
Method

We obtained horizontal and vertical strong motion and broadband records from the SCEC database, which archives data recorded by the Southern California Seismic Network (SCSN) and the Consortium of Organizations for Strong Motion Observation Systems (COSMOS) database, which archives strong motion data from the U.S. Geological Survey, California Geological Survey, and other strong motion arrays worldwide. For SCSN stations, we typically downloaded broadband velocity waveforms. If we found evidence of clipping (visual examination, or peak velocities exceeding 13 cm/s, the typical clip level of an STS-2 seismometer), we downloaded the accelerometer data. We performed gain and baseline corrections on the downloaded waveforms and integrated and/or differentiated to obtain acceleration, velocity, and displacement time histories. The displacement time histories obtained via integration were filtered using a 3-second, 4-pole high-pass Butterworth filter to reduce the influence of microseisms. We examined ground motions recorded at 150 Southern California Seismic Network (SCSN) stations located within 200 km epicentral distance of 70 Southern California events in the magnitude range M2.0 through M7.3. In addition to SCSN data, we also included strong motion records available from the COSMOS database from the 1989 M=7.0 Loma Prieta, 1991 M=5.8 Sierra Madre, 1992 M=7.3 Landers, 1992 M=6.4 Big Bear, and 1994 M=6.7 Northridge (and an M=5.1 aftershock) earthquakes. Ground motion envelopes were obtained from the 100- or 80-sample per second time histories by taking the maximum amplitudes over one-second windows.

Parameterization of ground motion envelopes

We used a parameterization that decomposes each observed ground motion envelope in our database into a combination of P-wave, S-wave, and ambient noise envelopes (Cua, 2005). The shape of each of the P- and S-wave envelopes with time is described by a rise time ($t_r$), constant amplitude ($A$) with an associated duration ($\Delta t$), and two decay parameters ($\tau$, $\gamma$).

\[
E_i = \begin{cases} 
0 & \text{for } t < T_i \\
\frac{A_i}{t_r_i}(t - T_i) & \text{for } T_i \leq t < T_i + t_r_i \\
A_i & \text{for } T_i + t_r_i \leq t < T_i + t_r_i + \Delta t_i \\
\frac{A_i}{(t - T_i - t_r_i - \Delta t_i + \tau_i)^\gamma} & \text{for } t \geq T_i + t_r_i + \Delta t_i 
\end{cases}
\]  

(1)

In Eqn. 1, the index $i = \{ \text{P-wave, S-wave} \}$, $t$ denotes time, and $T_i$ the phase arrival. The P-wave, S-wave, and ambient noise envelopes of a given record combine according to the rule:

\[
E_{\text{observed}} = \sqrt{E_P^2 + E_S^2 + E_{\text{ambient}}^2} + \varepsilon
\]  

(2)
where $E_{\text{observed}}$ is the observed ground motion envelope, $E_P$, $E_S$, and $E_{\text{ambient}}$ are the P-wave, S-wave, and ambient noise envelopes, and $\varepsilon$ is the difference between the observed and modeled envelope. Thus, an observed ground motion envelope can be described by 11 envelope parameters (5 each for the P- and S-wave envelopes, plus a constant for the ambient noise). For each of the 30,000 observed ground motion envelopes in our database, we use the Neighborhood Algorithm (Sambridge, 1999ab) to find the 11 maximum-likelihood envelope parameters that minimize $\varepsilon$ in Eqn.2. However, this set of 11 envelope parameters was not necessarily the only “good” solution for a given time history. There were likely numerous “good” solutions in the neighboring regions of the parameters space, due to the trade-offs between the rise time and duration parameters, as well as between the two coda decay parameters. We found that using a single parameter to describe the coda meant that we had to choose to either fit the amplitudes close to the peak, or those at larger times. Jennings et al (1968) also require 2 parameters to describe the coda decay. The peak P- and S-wave amplitude parameters did not trade-off with the other envelope parameters. The P- and S-wave amplitude relationships will be the focus of this paper.

Figure 1 shows the ground motion acceleration recorded at SCSN station Domenegoni Reservoir (DGR) during the 1994 M=6.7 Northridge earthquake. Figure 2 shows its ground motion envelope and the 11 maximum-likelihood envelope parameters from the Neighborhood Algorithm inversion. We developed attenuation relationships describing the magnitude and distance dependence of the 11 envelope parameters of 6 ground motion channels: vertical and horizontal acceleration, velocity, and (filtered) displacement. We adopted a binary (rock-soil) site classification and developed separate relationships for envelope parameters obtained from SCSN stations with NEHRP site class BC and above (rock) and C and below (soil).

**Site Classification**

The NEHRP-UBC site classification for each of the SCSN stations used in this study was obtained from the Southern California site classification map developed by Wills et al (2002). Wills et al (2002) created intermediate categories BC and CD to accommodate geologic units that had $V_{s30}$ values near the boundaries of the existing NEHRP-UBC site classes. In the binary site classification scheme adopted in our study, “rock” sites are those assigned to classes BC and above, and “soil” sites are those with classification C and below according to Wills et al (2002). Of the SCSN stations we used, 35 stations were classified as rock, and 129 stations were classified as soil sites. Separate attenuation relationships are developed for rock and soil sites. This allows us to investigate differences in the average properties of ground motions on rock and soil sites over the magnitude and distance ranges covered by this study. Since SCSN stations contribute the majority of the ground motions in our dataset, this study does not include records from very soft soils, or E class, or Bay mud-type sites.

Figure 1 shows the distribution of the data in magnitude and distance space for a given channel of ground motion. For each of 6 channels of ground motion (horizontal and
vertical acceleration, velocity, and filtered displacement), there are 958 records from rock sites, and 2,592 records from soil sites.

**Magnitude, distance, frequency band, and site-dependence of P- and S-wave amplitudes**

In this paper, we will focus our discussion on the behavior of P- and S-wave envelope amplitude parameters \( (A_P, A_S) \) as functions of magnitude, distance, site condition, and frequency. The functional form used to describe the magnitude, distance, and site dependence of the P- and S-wave envelope amplitudes is given below:

\[
\log_{10} A_{ij} = a_i M - b_i (R_1 + C_i (M)) - d_i \log_{10} (R_1 + C_i (M)) + e_{ij} + \varepsilon_i
\]  

where 

- \( i = 1, K, 24 \) (P-, S-wave amplitudes on rock and soil sites for 6 channels)
- \( j = 1, K, \) number of stations
- \( A_{ij} \) = "best" peak body wave amplitude for a given record from Neighborhood Algorithm inversion
- \( M \) = reported magnitude (\( M > 5 \) from SCSN)
- \( R \) = epicentral distance in km for \( M < 5 \), closest fault distance for \( M \geq 5 \) (when available)
- \( R_1 = \sqrt{R^2 + 9} \)
- \( C_i (M) = c_{1i} \exp(c_{2i} (M - 5)) \times \arctan(M - 5 + 1.4) \)
- \( \varepsilon_i \) = statistical or prediction error, : \( NID(0, \sigma^2) \)

The \( A_{ij} \)'s are the “best” peak body wave amplitudes from the Neighborhood Algorithm inversions on the individual ground motion envelopes. In the regressions to be discussed in the following section, these \( A_{ij} \)'s are referred to as “observations”. The 6 channels analyzed are vertical and root mean square horizontal components of ground motion acceleration, velocity, and filtered displacement. The \( A_{ij}, M, R, \) and \( \varepsilon_{ij} \) are \( (m \times 1) \) column vectors for the \( i^{th} \) amplitude parameter, where \( m \) is the number of data points available for the \( i^{th} \) amplitude parameter. For each of the \( i = 1, K, 24 \) amplitude parameters, Eqn.(2) is nonlinear in the unknowns, which are \( (a_i, b_i, c_{1i}, c_{2i}, d_i, e_{ij}) \).

Eqn.3 has strong influences from traditional strong motion attenuation relationships, in particular, from the work of Boore and Joyner (1982), Boore, Joyner, and Fumal (1993, 1997), and Campbell (1981, 2002). The physical motivations for the various terms are as enumerated by Boore and Joyner (1982) and Campbell (2002):

- \( \log_{10} A_{ij} \propto M \) is consistent with the basic definition of magnitude [Richter, 1935] as the logarithm of ground motion amplitude
- \( \log_{10} A_{ij} \propto \log_{10} R \) is consistent with the geometric attenuation of the seismic wavefront away from the source
• \( \log_{10} A_{ij} \propto R \) is consistent with anelastic attenuation due to material damping and scattering
• \( \log_{10} A_{ij} \propto e_{ij} \), where \( e_{ij} \) are station-specific site correction terms, is consistent with the multiplicative nature of site effects
• \( C_i(M) = c_{1i} \exp(c_{2i}(M - 5)) \times (\arctan(M - 5) + 1.4) \) is a magnitude-dependent saturation term that allows ground motion amplitudes at close distances to large earthquakes (\( M > 5 \)) to be relatively independent of magnitude. Ground motion simulations suggest that the shape of attenuation curves is magnitude-dependent, with ground motion amplitudes in the near-source region of large earthquakes approaching a limiting value (Hadley and Helmberger, 1980). Campbell (1981) found empirical evidence for such saturation in near-source peak accelerations from a dataset of near-source records (within 50 km) from global earthquakes with \( M > 5 \). Since our dataset spans a larger magnitude range (\( 2 < M \leq 7.3 \)), we modify Campbell’s original saturation term \( C_i(M) = c_{1i} \exp(c_{2i}M) \) with an \( (\arctan(M - 5) + 1.4) \) term to “turn on” saturation effects when \( M > 5 \), while allowing the logarithm of ground motion amplitudes to scale linearly with magnitude for \( M < 5 \). Figure 3 shows \( C(M) \) as a function of magnitude for various values of \( c1 \) (and \( c2 \) approximately 1). Values of \( c1 \) close to 0 mean no saturation, with increasing values of \( c1 \) indicating stronger saturation effects. \( C(M) \) has units of distance, and increasing \( C(M) \) increases the “effective epicentral distance” of a given station.

Due to the saturation function, \( C_i(M) \), Eqn.3 is a nonlinear function of the unknown model parameters \( (a, b, c_{1i}, c_{2i}, d_{ij}, e_{ij}) \). For each amplitude parameter \( i \), these model parameters are determined in a two-step process. In the first step, the Neighborhood Algorithm is used to find the set of model parameters – we drop the subscript \( i \) for brevity - \( (a, b, c_{1i}, c_{2i}, d_{ij}, e) \) that minimize the residual sum of squares, RSS, between the observed amplitudes \( A_{obs} \) and those predicted by the model \( A_{pred} \) from Eqn.3.

\[
RSS = \sum_{k=1}^{n} \left[ A_{obs} - A_{pred}(a, b, c_{1i}, c_{2i}, d_{ij}, e) \right]^2
\]

The station corrections, \( e_{ij} \), are obtained by averaging the residuals between model predictions and the observations available at a given station.

The standard error of regression, \( \sigma \), is a measure of how well the model fits the observations, and is given by

\[
\sigma = \sqrt{\frac{RSS}{ndof}}
\]

where \( ndof \) denotes the number of degrees of freedom, which equals the number of available observations, \( n \), less the number of model parameters determined via regression. Thus, without station corrections, our regressions have \( ndof = n - 6 \), while with station
corrections, the \( ndof=n-6\) \((\text{number of stations})\). Station corrections were calculated only if 3 or more records were available at a given station.

**Results**

We found the 11 best-fit envelope parameters (rise time, amplitude, duration, and 2 decay parameters for each of the P- and S-wave envelopes, and a constant for ambient noise) for each of the 30,000 ground motion envelopes in our database. We used the P- and S-wave envelope amplitudes obtained by fitting Eqn.1 to each of the 30,000 ground motion envelopes as the input ground motions; in the regression analyses, we refer to these P- and S-wave envelope amplitudes as observations, though they are in fact obtained via inversion. We used Eqn.3 as the functional form describing the magnitude, distance, and site dependence of various types of ground motion envelope amplitudes. We obtained unknown model parameters \((a, b, c_1, c_2, d, e)\) (Table 2) and the station corrections \(e_{ij}\) (available online at [http://resolver.caltech.edu/CaltechETD:etd-02092005-125601](http://resolver.caltech.edu/CaltechETD:etd-02092005-125601)) for 24 types of amplitude parameters (rms horizontal and vertical P- and S-wave acceleration, velocity, and displacement amplitudes on rock and soil sites) via the two-step process described previously.

Figure 3 shows the distance-dependence at various magnitudes of root mean square horizontal S-wave acceleration amplitudes on rock sites given by the attenuation model (without station corrections). The symbols are the observed amplitudes from which the model was derived. The fit of the attenuation model to the observations shown in Figure 3 is fairly representative of fits of the attenuation models for other envelope amplitude types to the available observations. In general, for \(M<5\), there is a strong dependence of log amplitude on log distance, due to geometric spreading. The slight curvature on the predicted attenuation curves at larger distances is due to the effect of the anelastic attenuation term. Saturation effects come into play at close distances to \(M>5\) events. Our attenuation relationships are indicative of a maximum median S-wave acceleration on the order of 0.5g.

Figure 4 shows the residuals, \(A_{\text{obs}} - A_{\text{pred}}(a, b, c_1, c_2, d, e)\), for horizontal S-wave acceleration amplitudes on rock sites as a function of magnitude and distance. In these plots, the solid line corresponds to a residual value of 0. The dashed lines correspond to the 95% confidence intervals, \(\pm 2\sigma\). There are no systematic trends in the residuals with either magnitude or distance. Such residual plots for other channels exhibit similar characteristics.

Figure 5 shows the corrections (in log units) for SCSN stations in this study located on rock sites (NEHRP site class BC and above) for rms horizontal S-wave acceleration amplitudes. Also shown are the numbers of records available at the stations, which are indicative of the statistical significance of the corresponding station corrections. Stations PAS, PFO, and ISA have corrections in excess of \(-0.3\) log units, translating to deamplification of greater than 50% relative to the average rock station. The number of records contributing to these corrections (50, 20, and 10 records, respectively) indicates
that these corrections are not likely due to randomness or chance, but rather, are evidence of consistent deamplification of rms horizontal S-wave accelerations at these sites. Incidentally, this approach allows us to define “average” rock stations whose observed ground motions are closest to those predicted by the best model (or whose station corrections are closest to 0). Some “average” rock stations over the time period 1998-2004 include GSC, PLM, HEC, EDW, and AGA. The set of stations considered “average” by this approach will evolve with time, depending on where seismic activity is concentrated over a given time period. Applying the station corrections on horizontal S-wave amplitudes results in a standard error of regression of $\sigma_{\text{corr}}=0.24$, a ~20% reduction relative to the standard error in the uncorrected case, $\sigma_{\text{uncorr}}=0.31$. This is manifested in the closer clustering of station-corrected predicted amplitudes about the true regression line in Figure 6.

**Discussion**

Using the attenuation models obtained (Table 2), we can compare how different channels of ground motion amplitudes vary as functions of magnitude and distance. We focus the discussion on general characteristics of, and differences between: 1) high and low frequency ground motions, 2) ground motions on rock versus soil sites, 3) horizontal versus vertical ground motion amplitudes, and 4) P- versus S-wave attenuation.

**High- versus low-frequency ground motions**

Acceleration, velocity, and displacement ground motions are often used as proxies for high-, mid-, and low-frequency ground motions. The S-wave envelope amplitude parameters are comparable to peak amplitudes when examining horizontal ground motion records. The saturation term $C(M)$ was designed to come into play at close distances to large events, with regression parameters $c_1$ and $c_2$ controlling the degree of magnitude-dependent saturation effects for $M>5$. Since $C(M)=0$ for $M<5$ for all components of ground motion (Figure 3), the coefficients $a$, $b$, and $d$ can be directly interpreted as the small magnitude ($M<5$) scaling factors for magnitude and distance dependence. Averaging coefficients $a$, $b$, and $d$ of rock and soil sites for horizontal acceleration, velocity, and displacement, small amplitude ground motions scale as follows:

\[
\begin{align*}
\text{horizontal S-wave acceleration, } u_S &\sim 10^{0.8M} \times 10^{-2.4 \times 10^{-3} R} \times \frac{1}{R^{1.4}} \\
\text{horizontal S-wave velocity, } u_S &\sim 10^{0.9M} \times 10^{-6.3 \times 10^{-4} R} \times \frac{1}{R^{1.5}} \\
\text{horizontal S-wave displacement, } u_S &\sim 10^{1.05M} \times 10^{-6.5 \times 10^{-7} R} \times \frac{1}{R^{1.5}} \\
\end{align*}
\]

(6)

In general, the geometric spreading term $\frac{1}{R^x}$ is fairly constant with frequency, with $x\sim 1.5$. The exponential decay term $10^{-yR}$ contributes to the distance decay of peak
acceleration for the distance range \((20 < R < 200 \text{ km})\), but has practically no contribution to the distance decay of peak velocity and displacement amplitudes within this distance range. This is consistent with high frequency ground motions being more sensitive to small scale crustal heterogeneities and thus exhibiting stronger scattering effects (Lay and Wallace, 1995), and studies of Hanks and McGuire (1981) showing that high frequency ground motions attenuate faster than lower frequencies.

Small-amplitude magnitude scaling from our regressions shows that the dependence of peak amplitudes on magnitude increases with decreasing frequency. This is consistent with Brune spectral scaling, where the high frequency amplitude spectrum scales with \(M_o^{13}\) and low frequency amplitude spectrum scales with \(M_0\). From simple scaling relations, we expect displacement amplitude \(u\) to scale with magnitude \(M\) as \(\log u \sim M\) at far field distances (several source dimensions away). This is consistent with the magnitude-dependence coefficients, \(a\), for the horizontal S-wave displacement envelope amplitudes for both rock and soil sites being close to 1 (Table 2).

At close distances to large, non-point source events \((M>6)\), we expect displacement amplitudes to scale as \(\log u \sim 0.5M\). Saturation effects are expected to be significant in this magnitude and distance range. We define “effective magnitude scaling” as the partial derivative of Eqn.3 with respect to \(M\), and use \(R=0\) and \(M=6\) to represent the condition “at close distances to large events”. The effective magnitude scaling takes into account the contributions of the saturation term \(C(M)\).

\[
\frac{\partial \log A}{\partial M} = a - b \left( \frac{c_1 \exp(c_2(M-5))}{1 + (M-5)^2} + C(M) \right) - d \left( \frac{c_1 \exp(c_2(M-5)) + C(M)}{R + c_1 \exp(c_2(M-5)) \ln(10)} \right)
\]

(7)

Evaluating Eqn.7 using the average \(a, b, d, c1, c2, e\) of rock and soil sites for horizontal S-wave displacement amplitudes and \(M=6, R=0\), we find that the effective magnitude scaling of displacement amplitudes at close distances to large events is \(\log u \sim 0.42M\), which is consistent with the expected scaling of \(\log u \sim 0.5M\) from simple scaling relations.

**Rock versus soil sites**

Magnitude-dependence and \(1/R\) distance attenuation are slightly stronger for ground motions on soil sites throughout the different frequency bands (Table 2). Saturation effects at close distances to large events are slightly stronger for ground motions recorded on soil sites, with the \(c1\) coefficient for soil always slightly larger than that for rock ground motions for a given channel. On average, ground motions on soil sites are twice as large as those on rock sites, since the regression coefficient \(e\) is consistently \(-0.3 \log \text{ (base10) units larger for soil than rock ground motions. However, ground motion amplification on soil sites relative to rock ground motions is actually
magnitude- and distance-dependent. Figure 6 shows S-wave amplitudes on rock and soil ground motions predicted by our attenuation relationships as functions of magnitude for different distance ranges for acceleration, velocity, and filtered displacement. Ground motion accelerations at close distances to large events exhibit the strongest saturation effects. Velocity and displacement ground motions also exhibit saturation, though to a lesser degree than that of acceleration. The saturation of amplitudes at close distances to large events appears to be primarily a source effect for velocity and displacement, since rock and soil sites appear equally affected. Horizontal acceleration is an exception, where the envelope attenuation relationships indicate a slight over-saturation of soil ground motions, which may be possibly attributed to non-linear site effects. This is consistent with the idea that nonlinear soil response contributes to ground motion saturation. At close distances to large events, the difference horizontal S-wave acceleration (or peak horizontal acceleration) between rock and soil ground motions decreases with increasing magnitude. This is consistent with the similarity in acceleration levels recorded at rock and soil sites subjected to strong ground motion observed by Campbell (1981). Across the different frequency bands, there is no difference between rock and soil ground motions at low amplitude levels (at large distances from small magnitude events).

**P- versus S-waves**

Eqn.3 was used to represent the magnitude- and distance-dependence of P-wave amplitudes, as it was for the S-wave amplitudes. The small magnitude scaling for P-waves is given by:

\[ u_P \sim 10^{0.7M} \times 10^{-4.1 \times 10^{-4} R} \times \frac{1}{R^{1.2}} \]

\[ u_P \sim 10^{0.8M} \times 10^{-4.3 \times 10^{-4} R} \times \frac{1}{R^{1.4}} \]

\[ u_P \sim 10^{0.9M} \times 10^{-1.0 \times 10^{-6} R} \times \frac{1}{R^{1.3}} \]

(8)

We can examine the relationships for vertical P-wave amplitudes and horizontal S-wave amplitudes to compare attenuation of peak P- and peak S-wave amplitudes. From Eqns.6 and 8, we can see that peak P-wave amplitudes have a weaker magnitude dependence as well as weaker 1/R decay when compared to peak S-wave amplitudes.

Peak P-wave amplitudes exhibit stronger saturation at close distances to large events than peak S-wave amplitudes on both rock and soil sites (Figure 7). The difference between P- and S-wave amplitudes at close distances to large events increases with decreasing frequency. This is consistent with P-waves having most energy in the high-frequency, and S-wave having most energy in the lower frequency range. The strong saturation of P-wave amplitudes may also be due to the difficulty in decomposing P- and S-waves at close distances when the time between the S- and P-wave arrivals is small.
Comparison with other ground motion attenuation relationships

Attenuation relationships are typically classified according to the tectonic environments from which they are derived and applicable: 1) shallow earthquakes from active tectonic regions, 2) stable continental regions, and 3) subduction zones. Since data from this study are almost exclusively from Southern California earthquakes, our relationships would be grouped in category 1. Some other commonly cited relationships in this category are Abrahamson and Silva (1997), Boore, Joyner, and Fumal (1997), and Campbell (1997). For conciseness, we will refer to these as AS97, BJF97, and C97, respectively.

The attenuation relationships presented in this study can be used to predict peak P- and S-wave amplitudes for horizontal and vertical acceleration, velocity, and (filtered) displacement for sites within 200 km of earthquakes in the magnitude range $2 < M < 7.3$. We used data from continuously recording strong and weak motion instruments from the SCSN network as well as triggered strong motion data from the COSMOS database. This allows our study to span a wider range of amplitudes than strong motion relationships derived from triggered accelerometers.

Only a small percentage of the records in our database lie in the region of magnitude-distance space typically covered by strong motion attenuation relationships. Each channel of ground motion we studied had 3,550 records; only 215 were within 80 km of events with $M>5$, which are the magnitude and distance limits used in BJF97. Thus, our envelope attenuation relationships are better-constrained by the data at lower amplitude levels (at intermediate and large distances from events with $M<5$), and less so in the magnitude-distance range typically spanned by strong motion attenuation relationships.

The ground motion acceleration levels predicted by the horizontal S-wave relations developed in this study are comparable with those of more traditional strong motion attenuation relationships. Figure 9 compares ground motion acceleration levels on rock for $M=5.5$ and $M=7$ predicted by our horizontal S-wave attenuation relationships with those predicted by AS97, BJF97 and C97 for peak ground acceleration. Our study did not take into account different styles of faulting. To evaluate BJF97, we used coefficients for unspecified faulting style; to evaluate AS97 and C97, we assumed a vertical strike-slip fault. The distance measure used we used was epicentral distance for events $M<5$, and closest fault distance (when available) for $M>5$. This is consistent with the Joyner and Boore distance, $r_{jb}$, for the magnitude range of comparison. The distance measure in C97 is $r_{seis}$, distance to the seismogenic part of the rupture. We use the recommended values in C97 for average depth to the seismogenic part of the rupture zone ($d_{seis}=6.2$ km for $M=5.5$, and $d_{seis}=3$ for $M=7$). The distance measure in AS97 is $r_{rup}$, closest distance to the rupture surface. We assume a surface-breaking vertical strike-slip fault, so that $r_{jb}$ and $r_{rup}$ are the same.

There is general agreement in the amplitude levels at close distances. AS97 and C97 amplitudes are consistently larger, since these relationships was derived from the arithmetic mean of 2 horizontal components, whereas our study and that of BJF97 used...
the geometric mean of 2 horizontal components. At fault distances greater than 10 km, our relationships, AS97, and C97 show similar distance decay. BJF97 amplitudes for M=5.5 are significantly larger at fault distances greater than 10 km. The differences in the predicted amplitude levels could perhaps be attributed to the differences in the datasets from which these relationships were obtained. Also, AS97, C97, and our relationships allow for magnitude-dependent scaling at short distances, whereas BJF97 has the same magnitude scaling at all distances. Joyner, Boore, and Fumal (1997) was derived from shallow, crustal western North America earthquakes and are to be used between M5.5 and M7.6. Campbell (1997) was derived from worldwide earthquakes of M>5 with distance to seismogenic rupture $R_{\text{seis}} < 60$ km in active tectonic regions. Boore, Joyner, and Fumal (1997) note that the amplitudes at large distances predicted by the BJF97 relationship are larger than those predicted by Joyner and Boore (1982), and attribute this difference to the difference in datasets, in particular, the contributions of the Loma Prieta, Petrolia, and Landers events, as well as improved quality control of the data (removing records that triggered on the S-wave, which may result in missing the peak ground motion).

In general, strong motion attenuation relationships are not as well constrained at larger distances, or have a limited distance range in which they are valid, since these relationships are typically obtained from triggered, strong motion instruments. In contrast, our relationships are obtained from a combination of broadband and strong motion data, and are thus better constrained at larger distances. For peak horizontal acceleration, it is reassuring that the amplitude levels at close distances to M>5 events predicted by our horizontal S-wave envelope amplitude relationships are comparable to those of BJF97, AS97, and C97, even though this region of our database is sparsely populated. There are significant differences between the amplitude levels at larger distances between our relationship and BJF97. However, we have confidence in the distance decay implied by our relationships at larger distances due to the large number of records in our dataset constraining the relationships in this distance range (see Figure 3).

Figure 10 compares peak vertical acceleration from AS97 and C97 with our relationships for vertical S-wave envelope amplitude. The amplitude levels are comparable for M=5.5, but our relationship shows very strong saturation at close distances for M=7. The difference may perhaps be attributed to the composition of the databases from which these relationships were derived. The strong saturation of our relationships is driven primarily by the records at Gilroy Array Station 1 and Gilroy Galivan College, with closest fault distances of 2.8 km and 3.0 km to the Loma Prieta earthquake.

We can compare our horizontal and vertical S-wave velocity amplitudes relationships with peak velocity amplitudes relationships from Campbell (1997). Figure 11 shows the comparison between our horizontal S-wave velocity envelope amplitude and peak horizontal velocity from C97. Figure 12 shows our vertical S-wave velocity envelope amplitude and peak vertical velocity from C97. (**There is an errata to Campbell (1997) published in 2003, and that may decrease the discrepancy between our velocity amplitude levels and Campbell (1997).)**
Conclusions

We used attenuation relationships for P- and S-wave amplitudes on rock and soil sites for horizontal and vertical acceleration, velocity, and filtered displacements to study 1) magnitude-dependent saturation of peak amplitudes on rock and soil sites in various frequency bands, 2) magnitude and distance scaling of P- and S-waves, and 3) the reduction of uncertainty in predicted ground motions due to the application of site-specific station corrections. We found that at the mid- and lower-frequency range, the saturation of peak ground motion amplitudes at close distances to large event is primarily a source effect, since rock and soil sites are equally affected. This is not the case for higher frequency ground motions (i.e., acceleration). It appears that peak ground acceleration on soil sites exhibit stronger saturation effects relative to rock sites at close distances to large events. This may be due to nonlinear site response. In general, the difference between ground motions amplitudes recorded on rock and soil sites decreases at very large amplitudes (at close distances to large events) and very small amplitudes (large distances from small events).

Our results indicate that P-wave amplitudes appear to exhibit stronger saturation than S-wave amplitudes at close distances to large events. This may be due to the inherent difficulty in decomposing P- and S-waves when the time between P- and S-wave arrivals is small.

We calculated station corrections for 150 SCSN stations for 6 channels of ground motion. The use of station-specific data can reduce the error between predicted and observed ground motions by about 20%.

We presented attenuation relationships for 6 channels of ground motion that are valid over a wider amplitude range than traditional strong motion attenuation relationships. While there are some discrepancies, in general, our relationships are consistent with the amplitude levels predicted by attenuation relationships for shallow crustal earthquakes in active tectonic regions.

Acknowledgements
(Add later)
Figure 1: Distribution in magnitude and distance space of data included in this study. Records were obtained from the SCEC and COSMOS databases. Records from stations located on sites with NEHRP site class BC and above are labeled “rock” records, while those recorded on sites with NEHRP site class C and below labeled as “soil” records.
Figure 2

- The upper graph shows the acceleration over time (cm/s²) with labeled time points T₀ and T₁.
- The lower graph includes various labeled points such as P-arrival, S-arrival, Es(t), As, trise, dls, γ₀, and γ₁.
- The equation for Es(t) is given as Es(t) = As \left( \frac{\tau_s}{(\tau_s + t_n)} \right)
- The notation γ₀ = \frac{t_n - T_n - \text{trise} - \text{dls}}{n}
- T₀ = phase arrival
- P-wave, S-wave

The diagrams illustrate seismic wave behavior and phase arrivals.
Figure 3

\[ C(M) = (\arctan(M-5) + 1.4) \times c \times \exp(c_2(M-5)) \]

the range of possible values of \( c_2 \) is limited to be close to 1

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Figure 3
Figure 6
Table 2
Attenuation relationships for ground motion envelope amplitudes

$$\log_{10} A = aM + b(R_1 + C(M)) + d(R_1 + C(M)) + e$$

$$R_1 = \sqrt{R + 9}$$

$$C(M) = c_1 \exp(c_2(M - 5)) \times (\arctan(M - 5) + 1.4)$$

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Figure 7
Figure 8
Figure 9
Figure 10
Figure 11
Figure 12
References


